A film model for free convection over a vertical porous plate with blowing or suction

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Abstract. A film model is described for free convective heat transfer and friction in the presence of wall suction or injection. The analysis yields a thermal correction factor, which appears to be the classical (Ackermann) expression, and a novel friction correction factor, derived here for the first time. Both correction factors are applied to free convection along a vertical flat porous plate for values of the Prandtl number of 0.73, 1 and 7. Subsequently, the film theory predictions are compared with the numerical and asymptotic results of a boundary layer analysis, performed by previous investigators. On the basis of this comparison, a modified correction factor is proposed which correlates excellently with the results of the boundary layer analyses.

**Ein Filmmodell für freie Konvektion
an einer senkrechten porösen Platte mit Ausblasung und Absaugung**

Zusammenfassung. Es wird ein Filmmodell beschrieben, das für freie Konvektion den Wärmeübergang und den Reibungsbeiwert bei Wandaus- oder -einblasung zu bestimmen gestattet. Die Untersuchung liefert einen thermischen Korrekturfaktor, der mit der klassischen Ackermann-Korrektur übereinstimmt und einen neuen, hier erstmalig abgeleiteten Korrekturfaktor für den Reibungsbeiwert. Beide Korrekturfaktoren werden zur Ermittlung der freien Konvektionsströmung entlang einer senkrechten, ebenen, porösen Platte herangezogen (für Prandtl-Zahlen: 0,73–1–7). Anschließend erfolgt ein Vergleich der Voraussagen nach der Filmtheorie mit den numerisch aus einer asymptotischen Grenzschichtanalyse erhaltenen Ergebnissen früherer Autoren. Aus diesem Vergleich resultiert ein modifizierter Korrekturfaktor für den Reibungsbeiwert, der eine ausgezeichnete Korrelation mit den Ergebnissen nach der Grenzschichttheorie liefert.

Symbols

c_p	[J kg^{-1} K^{-1}]	specific heat
E_t		relative difference in heat transfer between boundary layer and film model, Eq. (27)
E_u		relative difference in exerted friction between boundary layer and film model, Eq. (32)
f		dimensionless stream function
Gr_x		Grashof number, $g\,\beta\,(t_i - t_b)\,x^3/v^2$
g	[ms^{-2}]	gravitational acceleration
k	[Wm^{-1} K^{-1}]	thermal conductivity
L	[m]	plate length
n		power-law coefficient
Pr		Prandtl number, $v\,\varrho\,c_p/k$
t	[°C]	temperature

u	[ms^{-1}]	component of velocity in the x-direction
v	[ms^{-1}]	component of velocity in the y-direction
x	[m]	coordinate along the plate
y	[m]	coordinate normal to the plate

Greek symbols

β	[K^{-1}]	volumetric coefficient of thermal expansion
δ_t	[m]	film thickness
η		similarity variable ($= y\,(Gr_x/4)^{1/4}/x$)
Θ_t		ratio of heat transfer with and without wall transpiration
Θ_u		ratio of exerted friction with and without wall transpiration
Θ		dimensionless temperature ($= (t_b - t)/(t_b - t_i)$)
v	[m^2 s^{-1}]	kinematic viscosity
Ω		function for friction correction, Eq. (22)
ϱ	[kg m^{-3}]	density
ϕ_u		dimensionless frictional wall mass flux
ϕ_t		dimensionless thermal wall mass flux
$\varXi$		Ackermann function, Eq. (10)

Subscripts

asym	pertaining to asymptotic approximation
b	bulk or ambient fluid
blm	pertaining to boundary layer model
fm	pertaining to film model
i	wall

Superscripts

*	pertaining to zero suction
'	differentiation with respect to η
–	mean
+	modified film model

1 Introduction

For many years the classical film model correction factors have been used successfully to predict the effects of wall mass transfer on transport phenomena, such as skin friction and heat and mass transfer. The correction factors can be derived from a stagnant film analysis and applied to systems using either an imposed (wall suction or injection) mass flux or vapour(s) mass flux (condensation or evaporation). Recent

reviews of the film model are found in Bannwart [1], Bannwart and Bontemps [2], Brouwers and Chesters [3] and Brouwers [4]. The former two authors extended the model to include the effect of mass transfer on film thickness, the latter investigator added fog formation to the model.

The film model expressions have been applied mainly to forced convective transport phenomena. Until now, free convective friction with mass transfer has not been examined with the film model. This can probably be attributed to the fact that the classical film model correction factor for friction arises from a bulk velocity and hence is not particularly suited to free convection, where the ambient fluid is at rest. This classical correction factor was first derived by Mickley et al. [5]. It has been applied to forced convective flow through channels with porous walls by Brouwers and Chesters [3]. In the momentum equation for the film from which the forced convection friction correction factor was derived the buoancy and pressure gradient terms were omitted; though this neglect is allowable for forced convective flow, it is unacceptable for free convection.

Hence, in this note a stagnant film is considered in which buoyancy is retained in the momentum equation and an outer flow is absent. The solution of the energy equation for the film yields the temperature profile and thermal correction factor. Invoking the Oberbeck-Boussinesq approximations, the temperature profile in the film is inserted in the momentum equation. Solving this equation then yields the correction factor, derived here for the first time, for the effect of mass transfer on free convective friction. Subsequently, both the thermal and frictional correction factor are compared extensively with the theoretical results of Eichhorn [6] and Merkin [7, 21]. These latter results are based on a numerical and asymptotic boundary layer analysis of laminar free convection over a permeable vertical plate with wall transpiration. The application and comparison is restricted to laminar free convection because, to the author's knowledge, the literature on turbulent free convective flow with wall suction or injection is not yet available.

2 Film equations and derivation of correction factors

In this section the energy and momentum equations for a film in which there is an imposed induced velocity caused by wall injection or blowing, are derived and solved analytically. The solutions are employed to define the film model correction factors.

Consider a film, as depicted in Fig. 1, through which there is a transfer of heat and momentum in the y-direction, with gravity parallel to the wall in the downward x-direction. At the permeable wall ($y = 0$) the temperature is t_i. At a distance δ_t the bulk temperature t_b is attained. For the purposes of the present analysis the physical properties in the film are assumed to be constant, with the exception of the density, and the mixture is assumed to behave as an ideal and Newtonian gas.
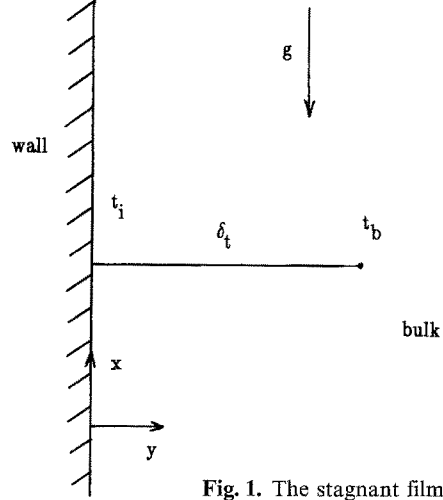

Fig. 1. The stagnant film

2.1 Energy equation

In order to investigate the effect of the induced velocity on heat transfer, attention is focused on the energy equation for the film. Considering a steady flow and neglecting variations of t in the x-direction, neglecting viscous dissipation, heat sources or radiation, this equation reads:

$$\varrho\, c_p v \frac{dt}{dy} = k \frac{d^2 t}{dy^2}, \tag{1}$$

subject to:

$$t\,(y = 0) = t_i, \tag{2}$$

$$t\,(y = \delta_t) = t_b. \tag{3}$$

Solution of Eq. (1), and application of boundary conditions (2) and (3) now produces a temperature profile in the film:

$$t\,(y) = (t_b - t_i)\left(\frac{e^{-\frac{y\phi_t}{\delta_t}} - 1}{e^{-\phi_t} - 1}\right) + t_i, \tag{4}$$

where ϕ_t represents the dimensionless thermal mass flux:

$$\phi_t = -\frac{\varrho\, v\, c_p \delta_t}{k}. \tag{5}$$

For a negligible induced velocity the temperature profile in the film is given simply by:

$$t\,(y)^* = (t_b - t_i)\frac{y}{\delta_t} + t_i, \tag{6}$$

which serves as reference level for the effect of the induced velocity on heat transfer. The asterisk represents the heat transfer in the case of zero mass transfer. The heat transfer from the fluid to the wall is described by Fourier's law as:

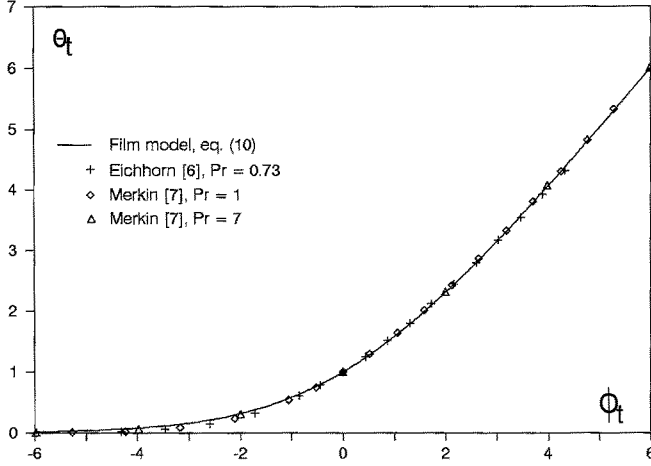$$q = k \frac{dt}{dy}\bigg|_{y=0}. \tag{7}$$

Fig. 2. The effect of transpiration on heat transfer, according to Eichhorn [6], Merkin [7] and the film model

The heat transfer from fluid to wall with an induced velocity is obtained by combining Eqs. (4) and (7):

$$q = \frac{k}{\delta_t}(t_b - t_i)\left(\frac{-\phi_t}{e^{-\phi_t} - 1}\right). \tag{8}$$

The heat transfer from fluid to wall without an induced velocity (i.e. $\phi_t \to 0$) is obtained by combining Eqs. (6) and (7):

$$q^* = \frac{k}{\delta_t}(t_b - t_i). \tag{9}$$

On comparing Eq. (8) with Eq. (9) the following film model correction factor is derived:

$$\Theta_{t,\mathrm{fm}} = \Xi(\phi_t) = \frac{-\phi_t}{e^{-\phi_t} - 1}. \tag{10}$$

This thermal correction factor $\Theta_{t,\mathrm{fm}}$ is the same as for forced convective heat transfer. It is commonly referred to as Ackermann correction, as it was first derived by Ackermann [8]. In Fig. 2 $\Theta_{t,\mathrm{fm}}$ is depicted as a function of ϕ_t.

2.2 Momentum equation

To examine the influence of the induced velocity on the wall shear stress, the momentum equation for the x-direction is investigated. Again neglecting gradients of properties in the x-direction this equation reads:

$$v\frac{du}{dy} = v\frac{d^2u}{dy^2} + \beta g(t - t_b), \tag{11}$$

with appropriate boundary conditions on u at the wall and bulk:

$$u(y = 0) = 0, \tag{12}$$

$$u(y = \delta_t) = 0. \tag{13}$$

In Eq. (11), in contrast to the conventional film model, the body forces are retained. These forces stem from variations in density caused by the temperature gradient across the film and in which the Oberbeck-Boussinesq approximations have been invoked (note that, though these approximations are generally attributed to Boussinesq [9], they were originally set out by Oberbeck [10], see the most interesting footnote of Joseph [11]). Boundary condition (13) reflects the state of rest of the ambient fluid beyond the thermal film. Furthermore, the integral approximations of Eckert [12] and Mabuchi [13] also arose from equal thermal and momentum boundary layer thickness. Eckert [12] obtained good agreements between experiments with air ($Pr \approx 0.7$) and an approximate integral method for impermeable plates. Mabuchi [13] also obtained good agreement between the integral methode and the numerical results ($Pr = 0.73$) of Eichhorn [6]. An analysis by Jaluria [14] revealed that the thermal and momentum boundary layer thicknesses are indeed of the same order of magnitude for $Pr \approx 1$.

Substituting Eq. (4) into Eq. (11), integrating the resulting equation twice with respect to y, and applying boundary conditions (12) and (13) results in:

$$u(y) = -\frac{g\beta\delta_t^2(t_b - t_i)}{v}\Theta_{t,\mathrm{fm}}\left(\frac{e^{-\frac{\phi_t y}{\delta_t}} - 1}{\phi_t\phi_u^2(1 - Pr)Pr} + \frac{y\,e^{-\phi_t}}{\delta_t\phi_t\phi_u}\right.$$
$$\left. - \left(\frac{e^{-\frac{\phi_u y}{\delta_t}} - 1}{e^{-\phi_u} - 1}\right)\left(\frac{e^{-\phi_u Pr} - 1}{\phi_t\phi_u^2(1 - Pr)Pr} + \frac{e^{-\phi_t}}{\phi_t\phi_u}\right)\right). \tag{14}$$

In this equation the Prandtl number Pr and frictional dimensionless mass flux:

$$\phi_u = -\frac{v\delta_t}{v}, \tag{15}$$

have been introduced. It is straightforward to see that the dimensionless frictional and thermal mass fluxes are related by:

$$\phi_t = \phi_u Pr. \tag{16}$$

Deriving Eq. (14) it has been assumed that $Pr \neq 1$. For $Pr = 1$ the velocity profile follows from Eqs. (4) and (11)–(13) as:

$$u(y) = -\frac{g\beta\delta_t^2(t_b - t_i)}{v}\Theta_{t,\mathrm{fm}}\left(\frac{y}{\delta_t\phi_u\phi_t}e^{-\frac{\phi_u y}{\delta_t}} + \frac{y}{\delta_t\phi_u\phi_t}e^{-\phi_t}\right.$$
$$\left. + \left(\frac{e^{-\phi_u} - e^{-\frac{\phi_u y}{\delta_t}}}{e^{-\phi_u} - 1} - 1\right)\left(\frac{e^{-\phi_u} + e^{-\phi_t}}{\phi_t\phi_u}\right)\right). \tag{17}$$

Without mass transfer the velocity profiles follow from Eqs. (5), (11) (with $v = 0$), (12) and (13) as:

$$u(y)^* = -\frac{g\beta\delta_t^2(t_b - t_i)}{v}\left(\frac{y^3}{6\delta_t^3} - \frac{y^2}{2\delta_t^2} + \frac{y}{3\delta_t}\right). \tag{18}$$

The film model correction factor for skin friction is now introduced by comparing the expression for the exerted friction with an induced velocity and that without.

The exerted shear stress by the fluid on the wall is governed by Newton's law as:

$$\tau = \eta \left. \frac{du}{dy} \right|_{y=0}. \tag{19}$$

Combining Eqs. (14) and (17) with Eq. (29) and application of Eq. (16) yields:

$$\tau = - g \varrho \beta \delta_t (t_b - t_i) \Theta_{t,\mathrm{fm}} \left(\frac{-1}{\phi_t \phi_u (1 - Pr)} + \frac{e^{-\phi_t}}{\phi_t \phi_u} \right.$$
$$\left. - \left(\frac{-1}{e^{-\phi_u} - 1} \right) \left(\frac{e^{-\phi_u Pr} - 1}{\phi_t \phi_u (1 - Pr) Pr} + \frac{e^{-\phi_t}}{\phi_t} \right) \right), \quad (Pr \neq 1),$$

$$\tau = - g \varrho \beta \delta_t (t_b - t_i) \Theta_{t,\mathrm{fm}} \left(\frac{1}{\phi_t \phi_u} + \frac{e^{-\phi_t}}{\phi_t \phi_u} \right. \tag{20}$$
$$\left. + \left(\frac{\phi_u}{e^{-\phi_u} - 1} \right) \left(\frac{e^{-\phi_u} + e^{-\phi_t}}{\phi_t \phi_u} \right) \right), \qquad (Pr = 1).$$

The exerted friction without an induced velocity follows from Eqs. (18) and (19):

$$\tau^* = - \frac{g \varrho \beta \delta_t (t_b - t_i)}{3}. \tag{21}$$

This expression is the limiting case of Eq. (20) for v (appearing in both ϕ_u and ϕ_t) tending to zero (the limit can be determined by substituting the third-order Taylor-MacLaurin expansion around zero of the exponential functions involved into equation (20)). Comparing Eq. (20) with Eq. (21) yields as correction factor:

$$\Theta_{u,\mathrm{fm}} = \Omega(\phi_u, Pr) = - \frac{3 \Xi (Pr \, \phi_u)}{Pr \, \phi_u^2}$$
$$\cdot \left(\frac{1 - \dfrac{\Xi(\phi_u)}{\Xi(Pr \, \phi_u)}}{1 - Pr} + (\Xi(\phi_u) - 1) \, e^{-Pr \phi_u} \right), \quad (Pr \neq 1),$$

$$\Theta_{u,\mathrm{fm}} = \Omega(\phi_u, Pr) = - \frac{3 \Xi(\phi_u)}{\phi_u^2} \left((2 \Xi(\phi_u) - 1) \, e^{-\phi_u} - 1 \right),$$
$$(Pr = 1), \tag{22}$$

where relations (10) and (16) have been inserted.

In Fig. 3 $\Theta_{u,\mathrm{fm}}$ has been depicted as a function of ϕ_u for $Pr = 0.73$, 1 and 7. Figure 3 illustrates that Eq. (22) predicts a friction reduction for both large suction and blowing rates. This in contrast to the correction factor for friction by forced convective flow, which predicts friction enhancement for suction and friction reduction for blowing (likewise the thermal correction), see Refs. [1–5]. Figure 3 also shows that $\Theta_{u,\mathrm{fm}}$ obtains a maximum for a negative ϕ_u when $Pr = 7$, that $\Theta_{u,\mathrm{fm}}$ is highest at $\phi_u = 0$ for $Pr = 1$, and that the correction factor reaches a maximum for a positive ϕ_u when $Pr = 0.73$. Replacing ϕ_u by $-\phi_u$ it can be proved with equation (22) that $\Theta_{u,\mathrm{fm}}$ is symmetrical with respect to the ordinate for $Pr = 1$.

In this section the temperature and velocity profiles in the film have been derived. On the basis of these solutions correction factors have been arrived at for the effect of mass
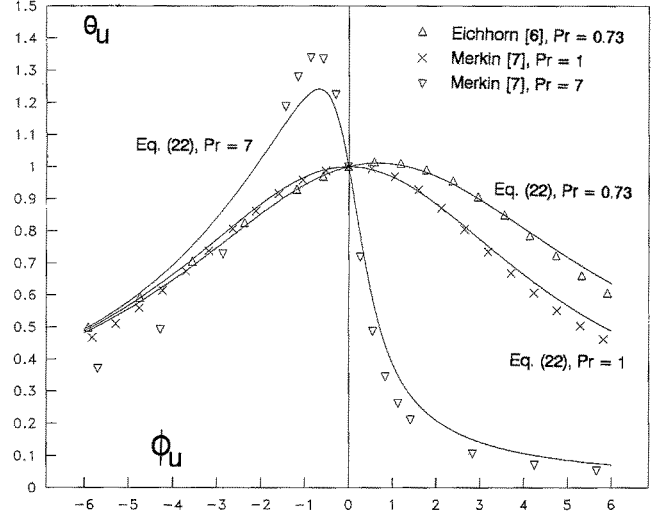

Fig. 3. The effect of transpiration on exerted friction, according to Eichhorn [6], Merkin [7] and the film model

transfer on free convective heat transfer and skin friction. In the next section these factors are applied to free convection and compared with numerical results obtained by previous investigators.

3 Application of film model and comparison with boundary layer model

The film model for the effect of mass transfer on both heat and momentum transfer is applied to free convective flow along an isothermal vertical plate. Subsequently the predictions of the model are compared with the results of Eichhorn [6] and Merkin [7]. These studies concern an analysis of laminar free convective boundary layer flow over a vertical plate with imposed transpiration. For the sake of completeness the studies in this field of Sparrow and Cess [15], Merkin [16], Parikh et al. [17] and Clarke [18] should also be mentioned.

Eichhorn [6] was the first to study the effect of suction and injection on free convective flow. Power law variations of wall temperature $(t_i - t_b = C_1 x^n)$ and transpiration velocity $(v = C_2 x^{(n-1)/4})$ were considered under which self-similar solutions of the governing equations are possible. For various wall stream function values, referred to as f_w, the dimensionless temperature gradient $-\Theta'(0)$ and velocity gradient $f''(0)$ at the wall were determined. This numerical boundary layer analysis provided results which can be used to validate the presented free convective film model.

3.1 Heat transfer

As mentioned above, Eichhorn [6] obtained $-\Theta'(0)$ for various f_w. The ratio of heat transfer with and without mass

Table 1. Values of $\Theta'(0)^*$ and $f''(0)^*$ for various Prandtl numbers according to Ostrach [19]

Pr	0.01	0.72	0.733	1	10	100	1000
$-\Theta'(0)^*$	0.0812	0.5046	0.5080	0.5671	1.1694	2.191	3.966
$f''(0)^*$	0.9862	0.6760	0.6741	0.6421	0.4192	0.2517	0.1450
$-\Theta'(0)^* f''(0)^*$	0.0801	0.3411	0.3424	0.3641	0.4902	0.5515	0.5751

Table 2. Numerical results of Eichhorn [6] and film model prediction of heat transfer ($n = 0$, $Pr = 0.73$)

Eichhorn [6] f_w	$-\Theta'(0)$	Eq. (23) $\Theta_{t,\text{blm}}$	Eq. (26) ϕ_t	Eq. (10) $\Theta_{t,\text{fm}}$	Eq. (27) E_t
−1.0	0.00748	0.0148	−4.320	0.0582	293.2%
−0.8	0.0264	0.0521	−3.456	0.113	116.9%
−0.6	0.0725	0.143	−2.592	0.210	46.9%
−0.4	0.162	0.320	−1.728	0.373	16.6%
−0.2	0.305	0.602	−0.864	0.629	4.5%
−0.1	0.399	0.787	−0.432	0.800	1.7%
0.0	0.507	1.000	0.000	1.000	0.0%
0.1	0.629	1.241	0.432	1.232	−0.7%
0.2	0.764	1.507	0.864	1.493	−0.9%
0.3	0.912	1.799	1.296	1.784	−0.8%
0.4	1.072	2.114	1.728	2.101	−0.6%
0.5	1.241	2.448	2.160	2.442	−0.2%
0.6	1.420	2.801	2.592	2.802	−0.04%
0.7	1.606	3.168	3.024	3.178	0.3%
0.8	1.798	3.546	3.456	3.569	0.6%
0.9	1.994	3.933	3.888	3.969	0.9%
1.0	2.192	4.323	4.320	4.378	1.2%

Table 3. Numerical results of Merkin [7] and film model prediction of heat transfer ($n = 0$, $Pr = 1$)

Merkin [7] f_w	$-\Theta'(0)$	Eq. (23) $\Theta_{t,\text{blm}}$	Eq. (26) ϕ_t	Eq. (10) $\Theta_{t,\text{fm}}$	Eq. (27) E_t
−1.0	0.0021	0.0037	−5.289	0.0268	624.3%
−0.8	0.0118	0.021	−4.231	0.0624	197.1%
−0.6	0.0465	0.082	−3.173	0.139	69.5%
−0.4	0.1341	0.236	−2.116	0.290	22.9%
−0.2	0.3023	0.533	−1.058	0.563	5.6%
−0.1	0.4223	0.745	−0.529	0.759	1.9%
0.0	0.5672	1.000	0.000	1.000	0.0%
0.1	0.7365	1.298	0.529	1.288	−0.8%
0.2	0.9290	1.638	1.058	1.621	−1.0%
0.3	1.1430	2.015	1.587	1.995	−1.0%
0.4	1.3763	2.426	2.116	2.406	−0.8%
0.5	1.6263	2.867	2.645	2.847	−0.7%
0.6	1.8901	3.332	3.173	3.312	−0.6%
0.7	2.1646	3.816	3.702	3.796	−0.5%
0.8	2.4468	4.313	4.231	4.293	−0.5%
0.9	2.7345	4.821	4.760	4.801	−0.4%
1.0	3.0260	5.335	5.289	5.316	−0.4%

transfer readily follows as:

$$\frac{\Theta'(0)}{\Theta'(0)^*} = \Theta_{t,\text{blm}}. \tag{23}$$

The film model correction factor can now be applied to free convective flow and compared with the heat transfer ratio (23) of the boundary layer model. To this end, the dimensionless mass flux ϕ_t and film thickness δ_t have to be assessed.

The heat flux is written in dimensionless form as:

$$q^* = -\frac{k}{x}\,\Theta'(0)^*(t_b - t_i)\left(\frac{Gr_x}{4}\right)^{1/4}, \tag{24}$$

where x is a coordinate along the wall. Ostrach [19] obtained values of $-\Theta'(0)^*$, which depend on Pr only, for various Prandtl numbers, here listed in Table 1. Combining Eqs. (9) and (24) yields as thermal film thickness:

$$\delta_t = -\frac{x}{\Theta'(0)^*}\left(\frac{Gr_x}{4}\right)^{-1/4}. \tag{25}$$

The thermal mass flux is related to f_w by:

$$\phi_t = -\frac{f_w(n+3)\,Pr}{\Theta'(0)^*}, \tag{26}$$

see Eqs. (5) and (25), and Eq. (11) from Ref. [6].

For $n = 0$ and $Pr = 0.73$ values of $\Theta'(0)$ were computed by Eichhorn [6] as a function of f_w, here summarized in Table 2. Table 2 also includes both ϕ_t, the pertaining film model correction factor $\Theta_{t,\text{fm}}$, $\Theta_{t,\text{blm}}$, and the relative error defined as:

$$E_t = \frac{\Theta_{t,\text{fm}} - \Theta_{t,\text{blm}}}{\Theta_{t,\text{blm}}}. \tag{27}$$

These data were already presented elsewhere (Ref. [20]). In order to validate the film model for other Pr numbers, Merkin [7] obtained values of $\Theta'(0)$ for $n = 0$ and both for $Pr = 1$ and $Pr = 7$. In Table 3 the results of $Pr = 1$ are listed and in Table 4 those pertaining to $Pr = 7$. In Fig. 2 the results are presented graphically. A glance at Table 3 shows that $-\Theta'(0)^*$ amounts to 0.5672 for $Pr = 1$, which is in good agreement with $\Theta'(0)^*$ of Table 1, provided by Ostrach [19]. This very small difference is an indication of the numerical accuracy. The tabulated values of E_t indicate that for suction, i.e. $f_w > 0$ and $\phi_t > 0$, the film model predictions correspond well to the numerical results, even for very large suction rates. For large injection rates, on the other hand, the deviation increases dramatically. However, the film model deviates less than 6% from the boundary layer model for $-0.864 \leq \phi_t \leq 4.320$. For this lowest value of ϕ_t injection already causes a 50% reduction in heat transfer. For this highest value of ϕ_t, suction enhances heat transfer by more than a factor of four.

Table 4. Numerical results of Merkin [7] and film model prediction of heat transfer ($n = 0$, $Pr = 7$)

Merkin [7] f_w	$-\Theta'(0)$	Eq. (23) $\Theta_{t,\mathrm{blm}}$	Eq. (26) ϕ_t	Eq. (10) $\Theta_{t,\mathrm{fm}}$	Eq. (27) E_t
−0.3	0.0072	0.0068	−5.976	0.015	120.6%
−0.2	0.0614	0.058	−3.984	0.076	31.0%
−0.1	0.3172	0.301	−1.992	0.315	4.7%
0.0	1.0543	1.000	0.000	1.000	0.0%
0.1	2.4416	2.316	1.992	2.301	−0.6%
0.2	4.2996	4.078	3.984	4.060	−0.4%
0.3	6.3347	6.008	5.976	5.991	−0.3%

3.2 Exerted friction

Eichhorn [6] and Merkin [7] obtained values of the dimensionless friction $f''(0)$ for various f_w. The ratio of actually exerted friction and friction without mass transfer follows as:

$$\frac{f''(0)}{f''(0)^*} = \Theta_{u,\mathrm{blm}}.\tag{28}$$

In order to validate the film model correction factor $\Theta_{u,\mathrm{fm}}$, provided by Eq. (22), it is applied to free convective flow and compared with Eq. (28). Accordingly, the dimensionless mass flux ϕ_u is determined with the help of Eqs. (16) and (26) as:

$$\phi_u = -\frac{f_w(n+3)}{\Theta'(0)^*}.\tag{29}$$

To free convective flow along an impermeable vertical plate applies:

$$\tau^* = 4\varrho\left(\frac{v}{x}\right)^2 f''(0)^*\left(\frac{Gr_x}{4}\right)^{3/4},\tag{30}$$

see Ostrach [19] or Eichhorn [6] among others. In Table 1 values of $f''(0)^*$ for various Prandtl numbers are listed, taken from Ostrach [19].

Combining Eqs. (21), (25) and (30) yields:

$$-f''(0)^* \, \Theta'(0)^* = \tfrac{1}{3}.\tag{31}$$

Table 1 illustrates that strictly speaking this equation is correct only for $Pr \approx 1$, so that the validity of the presented film model is limited to values of Pr which are about unity.

For $n = 0$ and $Pr = 0.73$ values of $f''(0)$ were computed by Eichhorn [6] as a function of f_w. In Table 5 these results are tabulated. This table also includes ϕ_u, the pertaining film model correction factor $\Theta_{u,\mathrm{fm}}$, $\Theta_{u,\mathrm{blm}}$, and the relative error defined as:

$$E_u = \frac{\Theta_{u,\mathrm{fm}} - \Theta_{u,\mathrm{blm}}}{\Theta_{u,\mathrm{blm}}}.\tag{32}$$

Tables 6 and 7 contain the results for $Pr = 1$ and $Pr = 7$ ($n = 0$), obtained by Merkin [7]. In Fig. 3 the results of the film model and boundary layer model are presented graphically.

Table 5. Numerical results of Eichhorn [6] and film model prediction of exerted friction ($n = 0$, $Pr = 0.73$)

Eichhorn [6] f_w	$f''(0)$	Eq. (28) $\Theta_{u,\mathrm{blm}}$	Eq. (29) ϕ_u	Eq. (22) $\Theta_{u,\mathrm{fm}}$	Eq. (32) E_u
−1.0	0.336	0.499	−5.917	0.487	−2.4%
−0.8	0.399	0.592	−4.734	0.579	−2.2%
−0.6	0.476	0.706	−3.550	0.691	−2.1%
−0.4	0.557	0.826	−2.367	0.815	−1.3%
−0.2	0.627	0.930	−1.183	0.929	−0.1%
−0.1	0.654	0.970	−0.592	0.972	−0.2%
0.0	0.674	1.000	0.000	1.000	0.0%
0.1	0.684	1.015	0.592	1.011	−0.4%
0.2	0.681	1.010	1.183	1.005	−0.5%
0.3	0.668	0.991	1.775	0.982	−0.9%
0.4	0.644	0.955	2.367	0.946	−0.9%
0.5	0.611	0.907	2.959	0.901	−0.7%
0.6	0.573	0.850	3.550	0.849	−0.1%
0.7	0.530	0.786	4.142	0.796	1.3%
0.8	0.488	0.724	4.734	0.742	2.5%
0.9	0.446	0.662	5.325	0.691	4.4%
1.0	0.409	0.607	5.917	0.642	5.8%

Table 6. Numerical results of Merkin [7] and film model prediction of exerted friction ($n = 0$, $Pr = 1$)

Merkin [7] f_w	$f''(0)$	Eq. (28) $\Theta_{u,\mathrm{blm}}$	Eq. (29) ϕ_u	Eq. (22) $\Theta_{u,\mathrm{fm}}$	Eq. (32) E_u
−1.1	0.2998	0.4668	−5.8181	0.5008	7.2%
−1.0	0.3275	0.5100	−5.2891	0.5424	6.4%
−0.9	0.3591	0.5592	−4.7602	0.5888	5.3%
−0.8	0.3946	0.6144	−4.2313	0.6401	4.2%
−0.7	0.4333	0.6747	−3.7024	0.6957	3.1%
−0.6	0.4740	0.7381	−3.1735	0.7544	2.2%
−0.5	0.5148	0.8072	−2.6446	0.8140	0.8%
−0.4	0.5536	0.8620	−2.1157	0.8715	1.1%
−0.3	0.5879	0.9154	−1.5867	0.9230	0.8%
−0.2	0.6154	0.9583	−1.0578	0.9641	0.6%
−0.1	0.6341	0.9874	−0.5289	0.9908	0.3%
0.0	0.6422	1.0000	0.0000	1.0000	0.0%
0.1	0.6386	0.9944	0.5289	0.9908	−0.4%
0.2	0.6229	0.9699	1.0578	0.9641	−0.6%
0.3	0.5959	0.9279	1.5867	0.9230	−0.5%
0.4	0.5596	0.8714	2.1157	0.8715	0.01%
0.5	0.5172	0.8054	2.6446	0.8140	1.1%
0.6	0.4726	0.7359	3.1735	0.7544	2.5%
0.7	0.4292	0.6683	3.7024	0.6957	4.1%
0.8	0.3894	0.6064	4.2313	0.6401	5.6%
0.9	0.3540	0.5512	4.7602	0.5888	6.8%
1.0	0.3232	0.5033	5.2891	0.5424	7.8%
1.1	0.2965	0.4617	5.8181	0.5008	8.5%

Tables 5 and 6 and Fig. 3 illustrate the close agreement between both models for $Pr \approx 1$. Even for very large negative and positive values of ϕ_u (i.e. strong blowing and suction) the boundary layer model results approximate the film model lines very well. Tables 5 and 6 reveal that the film model deviates less than 8.5% from the boundary layer model for $|\phi_u| \leq 5.8181$ and $Pr = 0.73$ and $Pr = 1$. For $Pr = 7$, on the other hand, a great discrepancy between both models is

Table 7. Numerical results of Merkin [7] and film model prediction of exerted friction ($n = 0$, $Pr = 7$)

Merkin [7] f_w	$f''(0)$	Eq. (28) $\Theta_{u,\text{blm}}$	Eq. (29) ϕ_u	Eq. (22) $\Theta_{u,\text{fm}}$	Eq. (32) E_u
−2.0	0.1665	0.3693	−5.6910	0.5169	40.0%
−1.5	0.2217	0.4918	−4.2682	0.6600	34.2%
−1.0	0.3284	0.7285	−2.8455	0.8677	19.1%
−0.5	0.5350	1.1868	−1.4227	1.1399	−4.0%
−0.4	0.5767	1.2793	−1.1382	1.1924	−6.8%
−0.3	0.6039	1.3393	−0.8536	1.2319	−8.0%
−0.2	0.6017	1.3347	−0.5691	1.2372	−7.3%
−0.1	0.5522	1.2249	−0.2845	1.1697	−4.5%
0.0	0.4508	1.0000	0.0000	1.0000	0.0%
0.1	0.3239	0.7185	0.2845	0.7730	7.6%
0.2	0.2193	0.4865	0.5691	0.5749	18.2%
0.3	0.1554	0.3447	0.8536	0.4371	26.8%
0.4	0.1181	0.2620	1.1382	0.3465	32.3%
0.5	0.0949	0.2105	1.4227	0.2852	35.5%
1.0	0.0474	0.1051	2.8455	0.1491	41.9%
1.5	0.0317	0.0703	4.2682	0.1002	42.5%
2.0	0.0238	0.0528	5.6910	0.0753	42.6%

observed. In the next section attention is paid to the effect of extremely large blowing and suction on heat transfer and exerted friction.

4 Asymptotic analysis

When attention is paid to the film model and boundary layer model results shown in Figs. 2 and 3, the question arises whether there are limiting values of $\Theta_{t,\text{fm}}$ and $\Theta_{u,\text{fm}}$ for large (positive or negative) ϕ_t and ϕ_u. In this section this supposition is examined in some detail. To this end, $\Theta_{t,\text{fm}}$ and $\Theta_{u,\text{fm}}$ are analyzed for large $|\phi_t|$ and $|\phi_u|$, respectively, and compared with the asymptotic results of Refs. [6, 21] for large blowing and suction. These asymptotic expressions were derived from the boundary layer equations and are valid and applicable for all values n and $Pr > 0$.

For large positive ϕ_t Eq. (10) yields:

$$\Theta_{t,\text{fm}} = \Xi(\phi_t) \approx \phi_t \quad (\phi_t \to \infty), \tag{33}$$

as $e^{-\phi_t}$ tends to zero. This asymptotic expression for large suction has also been found by Eichhorn [6]; from Eq. (17) of this paper $\Theta'(0)$ can be derived and combined with Eqs. (23) and (26) to obtain expression (33).

For very large negative ϕ_t Eq. (10) tends to:

$$\Theta_{t,\text{fm}} = \Xi(\phi_t) \approx -\phi_t e^{\phi_t} \quad (\phi_t \to -\infty). \tag{34}$$

For this extreme case no asymptotic expression of $\Theta'(0)$ has been derived yet from the boundary layer equations.

In the following $\Theta_{u,\text{fm}}$ is examined for large suction and blowing. For large positive ϕ_u it can be verified (with the help of Eq. (33)) that Eq. (22) behaves like:

$$\Theta_{u,\text{fm}} \approx \frac{3}{Pr\,\phi_u} \quad (\phi_u \to \infty). \tag{35}$$

This relation is in accordance with the asymptotic expansion of Merkin [21] for large suction:

$$\Theta_{u,\text{asym}} \approx -\frac{1}{\phi_u\,Pr\,f''(0)^*\,\Theta'(0)^*} + O(\phi_u^{-3}) \quad (\phi_u \to \infty), \tag{36}$$

for $f''(0)^*\,\Theta'(0)^* = -\frac{1}{3}$, thus for $Pr \approx 1$ only, see Table 1. For large blowing Eq. (22) behaves like:

$$\Theta_{u,\text{fm}} \approx -\frac{3}{\phi_u} \quad (\phi_u \to -\infty). \tag{37}$$

The three lines in Fig. 3 indeed tend to the same value of $\Theta_{u,\text{fm}}$ for $\phi_u \to -\infty$, irrespective the value of Pr. Furthermore, the value of $\Theta_{u,\text{fm}}$ as such amounts to about 0.5 for $\phi_u = -6$, which is in quantitative agreement with Eq. (37). Relation (37) also agrees with the asymptotic result of Ref. [21]:

$$\Theta_{u,\text{asym}} \approx \frac{1}{\phi_u\,f''(0)^*\,\Theta'(0)^*} \quad (\phi_u \to -\infty), \tag{38}$$

for $f''(0)^*\,\Theta'(0)^* = -\frac{1}{3}$, thus again for $Pr \approx 1$.

So, by definition, the film model yields a value of unity for the correction factors for zero mass transfer. Furthermore, in this section it has been demonstrated that the film model predictions of heat transfer with large suction coincide with the results of the asymptotic analyses of the boundary layer equations by Refs. [6, 21]. For large blowing an asymptotic approximation of the heat transfer has not, as yet, been derived from the boundary layer model. The difference between $\Theta_{t,\text{fm}}$ and numerical results, however, increases with increasing negative ϕ_t, see Tables 2–4. Hence it is concluded that the heat transfer for large negative ϕ_t will not follow Eq. (34). The film model predictions of exerted friction agree with the asymptotic analysis of the boundary layer equations by Ref. [21] for $Pr \approx 1$ only, both for large suction and blowing.

In order to fit the film model to the boundary layer model for all Pr, the following modified correction factor is suggested:

$$\Theta_{u,\text{fm}}^+ = \Omega(\phi_u^+, Pr), \tag{39}$$

with ϕ_u^+ as stretched dimensionless transpiration rate:

$$\phi_u^+ = -3 f''(0)^*\,\Theta'(0)^*\,\phi_u = 3(n+3) f_w f''(0)^*, \tag{40}$$

see Eqs. (22) and (26). For large positive and negative ϕ_u^+ and all values of Pr the correction factor $\Theta_{u,\text{fm}}^+$ matches Eqs. (36) and (38). In Tables 8–10 the results of Refs. [6, 21] are summarized (the same as in Tables 5–7). In these tables ϕ_u^+ and $\Theta_{u,\text{fm}}^+$ following Eqs. (22), (39) and (40) are included as well. The results illustrate that, generally speaking, the agreement between modified film model and boundary layer model is augmented for $Pr = 0.73$, 1 and 7. For all these Prandtl numbers the film model now matches the boundary layer results excellently. This feature of the modified film model is
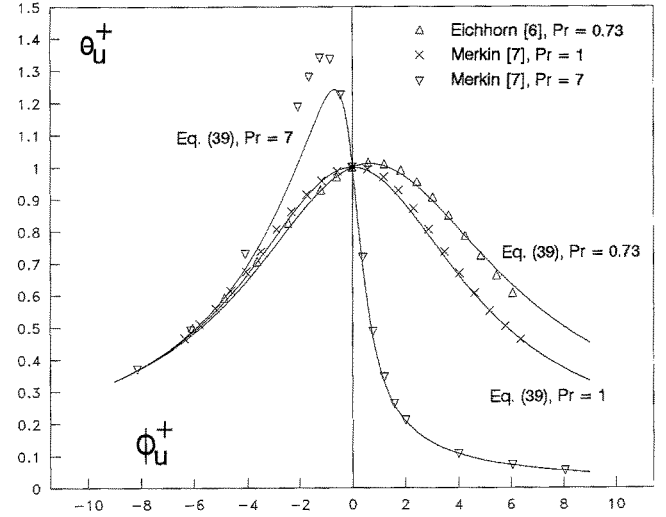
Table 8. Numerical results of Eichhorn [6] and modified film model prediction of exerted friction ($n = 0$, $Pr = 0.73$)

Eichhorn [6] f_w	$f''(0)$	Eq. (28) $\Theta_{u,\text{blm}}$	Eq. (40) ϕ_u^+	Eq. (39) $\Theta_{u,\text{fm}}^+$	Eq. (32) E_u
-1.0	0.336	0.499	-6.066	0.477	-4.4%
-0.8	0.399	0.592	-4.853	0.569	-3.8%
-0.6	0.476	0.706	-3.640	0.682	-3.4%
-0.4	0.557	0.826	-2.426	0.809	-2.1%
-0.2	0.627	0.930	-1.213	0.926	-0.4%
-0.1	0.654	0.970	-0.607	0.971	-0.1%
0.0	0.674	1.000	0.000	1.000	0.0%
0.1	0.684	1.015	0.607	1.011	-0.4%
0.2	0.681	1.010	1.213	1.004	-0.6%
0.3	0.668	0.991	1.820	0.980	-1.1%
0.4	0.644	0.955	2.426	0.942	-1.4%
0.5	0.611	0.907	3.033	0.894	-1.4%
0.6	0.573	0.850	3.640	0.841	-0.1%
0.7	0.530	0.786	4.246	0.786	0.0%
0.8	0.488	0.724	4.853	0.732	1.1%
0.9	0.446	0.662	5.459	0.680	2.7%
1.0	0.409	0.607	6.066	0.631	4.0%

Fig. 4. The effect of transpiration on exerted friction, according to Eichhorn [6], Merkin [7] and the modified film model

Table 9. Numerical results of Merkin [7] and modified film model prediction of exerted friction ($n = 0$, $Pr = 1$)

Merkin [7] f_w	$f''(0)$	Eq. (28) $\Theta_{u,\text{blm}}$	Eq. (40) ϕ_u^+	Eq. (39) $\Theta_{u,\text{fm}}^+$	Eq. (32) E_u
-1.1	0.2998	0.4668	-6.3578	0.4631	-0.8%
-1.0	0.3275	0.5100	-5.7798	0.5036	-1.3%
-0.9	0.3591	0.5592	-5.2018	0.5497	-1.7%
-0.8	0.3946	0.6144	-4.6238	0.6016	-2.1%
-0.7	0.4333	0.6747	-4.0459	0.6592	-2.3%
-0.6	0.4740	0.7381	-3.4679	0.7214	-2.3%
-0.5	0.5148	0.8072	-2.8899	0.7864	-2.6%
-0.4	0.5536	0.8620	-2.3119	0.8507	-1.3%
-0.3	0.5879	0.9154	-1.7339	0.9096	-0.8%
-0.2	0.6154	0.9583	-1.1560	0.9575	-0.8%
-0.1	0.6341	0.9874	-0.5780	0.9890	1.6%
0.0	0.6422	1.0000	0.0000	1.0000	0.0%
0.1	0.6386	0.9944	0.5780	0.9890	-0.5%
0.2	0.6229	0.9699	1.1560	0.9575	-1.3%
0.3	0.5959	0.9279	1.7339	0.9096	-1.8%
0.4	0.5596	0.8714	2.3119	0.8507	-2.1%
0.5	0.5172	0.8054	2.8899	0.7864	-2.4%
0.6	0.4726	0.7359	3.4679	0.7214	-2.0%
0.7	0.4292	0.6683	4.0459	0.6592	-1.4%
0.8	0.3894	0.6064	4.6238	0.6016	-0.8%
0.9	0.3540	0.5512	5.2018	0.5497	-0.3%
1.0	0.3232	0.5033	5.7798	0.5036	0.1%
1.1	0.2965	0.4617	6.3578	0.4631	0.3%

Table 10. Numerical results of Merkin [7] and modified film model prediction of exerted friction ($n = 0$, $Pr = 7$)

Merkin [7] f_w	$f''(0)$	Eq. (28) $\Theta_{u,\text{blm}}$	Eq. (40) ϕ_u^+	Eq. (39) $\Theta_{u,\text{fm}}^+$	Eq. (32) E_u
-2.0	0.1665	0.3693	-8.1144	0.3688	-0.1%
-1.5	0.2217	0.4918	-6.0858	0.4861	-1.2%
-1.0	0.3284	0.7285	-4.0572	0.6863	-5.4%
-0.5	0.5350	1.1868	-2.0286	1.0192	-14.1%
-0.4	0.5767	1.2793	-1.6229	1.1002	-14.0%
-0.3	0.6039	1.3393	-1.2172	1.1786	-12.0%
-0.2	0.6017	1.3347	-0.8114	1.2355	-7.4%
-0.1	0.5522	1.2249	-0.4057	1.2103	-1.2%
0.0	0.4508	1.0000	0.0000	1.0000	0.0%
0.1	0.3239	0.7185	0.4057	0.6816	-5.1%
0.2	0.2193	0.4865	0.8114	0.4541	-6.7%
0.3	0.1554	0.3447	1.2172	0.3271	-5.1%
0.4	0.1181	0.2620	1.6229	0.2532	-3.4%
0.5	0.0949	0.2105	2.0286	0.2059	-2.2%
1.0	0.0474	0.1051	4.0572	0.1053	-0.02%
1.5	0.0317	0.0703	6.0858	0.0704	-0.1%
2.0	0.0238	0.0528	8.1144	0.0528	0.0%

5 Mean heat transfer and friction

In the above attention has been focused on the effect of mass transfer on local heat transfer and exerted friction. In the following the effect on overall (and mean) heat transfer and friction is analyzed in some detail.

The mean heat transfer follows from:

$$\bar{q} = \frac{1}{L} \int_{x=0}^{L} q \, dx, \tag{41}$$

with:

$$q = \Theta_{t,\text{fm}} q^* = -\Theta_{t,\text{fm}} \frac{k}{x} \Theta'(0)^* (t_b - t_i) \left(\frac{Gr_x}{4} \right)^{1/4}, \tag{42}$$

also confirmed by Fig. 4 where $\Theta_{u,\text{fm}}^+$ is depicted against ϕ_u^+ and the results of Refs. [6, 21] included. Summarizing, from the film model and asymptotic analysis of Ref. [21] a modified film model correction factor $\Theta_{u,\text{fm}}^+$ is constructed which is recommended for future computations of free convective friction in the presence of wall transpiration.

see Eqs. (10) and (24). The mean exerted friction is defined as:

$$\bar{\tau} = \frac{1}{L} \int_{x=0}^{L} \tau \, dx, \tag{43}$$

with:

$$\tau = \Theta_{u,\text{fm}}^{+} \tau^{*} = \Theta_{u,\text{fm}}^{+} \, 4\varrho \left(\frac{v}{x}\right)^{2} f''(0)^{*} \left(\frac{Gr_{x}}{4}\right)^{3/4}, \tag{44}$$

see Eqs. (22), (30) and (39). For free convection with mass transfer, i.e. $\Theta_{t,\text{fm}} \neq 1$ and $\Theta_{u,\text{fm}}^{+} \neq 1$, the integrals in Eqs. (41) and (43) can, in general, not be solved in closed form, since both ϕ_{t} and ϕ_{u}^{+} depend on x. The similarity solution of the equations, however, permits an analytic treatment of the integrals. In this case ϕ_{t} and ϕ_{u}^{+}, and consequently $\Theta_{t,\text{fm}}$ and $\Theta_{u,\text{fm}}^{+}$, are constant, see Eqs. (10), (22), (26) and (29). Equations (41) and (42) then yield:

$$\bar{q} = -\Theta_{t,\text{fm}} \frac{4k}{3L} \Theta'(0)^{*} (t_{b} - t_{i}) \left(\frac{Gr_{L}}{4}\right)^{1/4}, \tag{45}$$

and Eqs. (43) and (44):

$$\bar{\tau} = \Theta_{u,\text{fm}}^{+} \frac{16}{5} \varrho \left(\frac{v}{L}\right)^{2} f''(0)^{*} \left(\frac{Gr_{L}}{4}\right)^{3/4}. \tag{46}$$

Hence, in the special case of constant ϕ_{t} and ϕ_{u}^{+} the mean heat transfer and friction simply follow from multiplying the mean zero mass transfer levels (which are usually documented in the literature) by the – constant – film model correction factors.

6 Conclusions

In this note film model correction factors have been derived for the effect of mass transfer on free convective heat transfer and wall friction along a vertical plate. First, on the basis of the energy equation for a stagnant film the temperature distribution and thermal correction factor $\Theta_{t,\text{fm}}$ have been derived. Subsequently, the momentum equation for the film was derived in which, unlike the conventional film model, the buoyancy terms have been retained. Invoking the Oberbeck-Boussinesq approximations and substitution of the temperature profile resulted in the velocity profile, the exerted friction and an analytical expression for frictional correction factor $\Theta_{u,\text{fm}}$.

This novel film model correction factor reveals the major effect of the Prandtl number on friction. Furthermore, it predicts that free convective friction is reduced by both large suction and injection. Next, this frictional correction factor and the thermal correction factor are compared with theoretical results of Refs. [6, 21] for a broad range of transpiration levels. These studies concerned a boundary layer model of free convection along an isothermal vertical plate whereby the governing equations were solved numerically, the Prandtl number ranging from 0.73 to 7.

The thermal correction factor agreed with the literature within 6% for $-0.864 \leq \phi_{t} \leq 4.320$ for $Pr = 0.73$, 1 and 7. For this range of ϕ_{t} the basic film model is well suited to describe the effect of mass transfer on free convective heat transfer. The frictional correction factor appeared to agree within 8.5% with the boundary layer model for $|\phi_{u}| \leq 5.8181$ and $Pr = 0.73$ and 1. For $Pr = 7$ deviations of more than 10% were found.

The behaviour of both $\Theta_{t,\text{fm}}$ and $\Theta_{u,\text{fm}}$ for large $|\phi_{t}|$ and $|\phi_{u}|$, respectively, have been examined in detail. Comparing the limiting values of $\Theta_{t,\text{fm}}$ with the asymptotic results of Refs. [6, 21] revealed that the they coincide for large positive ϕ_{t} only. A comparison between $\Theta_{u,\text{fm}}$ and the asymptotic results of Ref. [21] showed that they are practically identical for $Pr \approx 1$ and large $|\phi_{u}|$. On the basis of this insight a modified correction factor $\Theta_{u,\text{fm}}^{+}$ has been introduced which correlates well with the boundary layer model for large $|\phi_{u}^{+}|$ and irrespective the value of Pr.

In the film model considered the induced velocity stems from imposed wall injection or suction. Similar corrections will follow, however, when this velocity originates from a condensing or evaporating vapours. In a previous film analysis of a binary mixture, consisting of a vapour and stagnant noncondensables (see Refs. [3, 4]), we have seen namely hat the diffusional velocity is also constant across the film, likewise suction/injection.

The only condition for this generalization to be true is that the densities of the diffusing and stagnant components are identical. If this is not the case buoyancy driven flow in the film is, besides the temperature gradient, also introduced by the variation in composition of the film. Hence, transport phenomena are then independently affected by the induced velocity and buoyancy, both originating from a concentration gradient across the film.

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