An improved tangency condition for fog formation in cooler-condensers

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Abstract—In 1950 Johnstone *et al.* (*Ind. Engng Chem.* **42**, 2298–2302 (1950)) introduced the tangency condition to determine fog (or mist) formation in binary mixtures in cooled channels. In the present analysis it is demonstrated that their condition is erroneous and an improved equation is derived. The condition is based on the heat and diffusional mass transfer rates to a condenser wall, and the slope of the saturation line of the vapour component at the wall temperature. The transfer rates follow from a thorough analysis of the energy and diffusion equation in a stagnant film next to the wall (the classical 'film model' or 'film theory'). The validity of the improved tangency condition is assessed against wall condensation experiments by Johnstone *et al.*, concerning mixtures of nitrogen with vapours of water and n-butyl alcohol, yielding satisfactory agreement.

1. INTRODUCTION

A PIONEERING article on fog formation in mixtures of noncondensables and vapour in cooled channels has been published by Johnstone *et al.* [1]. On the basis of a film model analysis the tangency condition was derived to determine the critical wall temperature for fog formation in flowing binary mixtures in the presence of wall condensation. This condition followed from a consideration of the vapour pressure and temperature gradients at the wall, which were compared with the slope of the saturation line at the wall temperature. Until now the condition has been used unaltered to examine fog formation, see the extensive reviews by Sekulić [2] and Koch [3] of subsequent literature in the field of fog formation.

Experiments were furthermore performed by Johnstone *et al.* [1] with mixtures of nitrogen and vapours of sulphur, n-butyl alcohol and water to validate the derived condition. Two experimental departures from the theoretical predictions of (no) fog formation were observed, namely:

(a) no fog formation, though predicted, and,
(b) fog formation, though superheating was proved theoretically.

The former deviation could be attributed to the absence of sufficient nuclei in the gas flow, although in the examined mixtures extra nuclei were generated artificially. The second discrepancy, only found with film condensation of n-butyl alcohol and dropwise condensation of water, could not be explained satisfactorily.

In this paper it is shown that Johnstone *et al.* [1] employed an erroneous equation to investigate fog formation; the improved tangency equation will be derived here. Furthermore, the nitrogen–water vapour condensation experiments of Johnstone *et al.* [1] are found to correlate excellently with this improved condition.

2. BASIC EQUATIONS OF THE FILM AND THEIR SOLUTIONS

In this section the profiles of the vapour mole fraction and temperature in a stagnant film are derived. The required basic equations of diffusion and energy can be found in Bird *et al.* [4].

In the film—a steady-state system in which variations in the x-direction (which will later be identified with the direction of flow in a channel) are neglected—the local mass balance equation, see Fig. 1, is

$$v\rho^+ \frac{dc^+}{dy} = \rho^+ \mathbb{D} \frac{d^2 c^+}{dy^2} \tag{1}$$

where v is given by

$$v = -\frac{\mathbb{D}}{1-c^+} \frac{dc^+}{dy}. \tag{2}$$

This velocity is traditionally referred to as 'convective velocity', 'bulk flow', or 'Stefan flow'. It is induced by the flow of vapour through the mixture and plays an important role in mixtures with a high vapour mole fraction. The boundary conditions on c^+, see Fig. 1, are

$$c^+(y=0) = c_i^+ \tag{3}$$

$$c^+(y=\delta_c) = c_b^+ \tag{4}$$

where c_i^+ is the vapour mole fraction at the interface and c_b^+ the mole fraction of the bulk. Substituting equation (2) in equation (1), solving the resulting

NOMENCLATURE

c^+ vapour mole fraction

c_p^+ molar specific heat [J kmol^{-1} K^{-1}]

$\mathbb{D}$ diffusion coefficient [m^2 s^{-1}]

D_h hydraulic diameter [m]

F^+ saturation function, see equation (14)

G^+ relation between c^+ and t in the superheated region

g mass transfer coefficient [kmol m^{-2} K^{-1}]

h heat transfer coefficient [W m^{-2} K^{-1}]

k thermal conductivity [W m^{-1} K^{-1}]

Le Lewis number, $k/\rho^+ c_p^+ \mathbb{D}$

Nu Nusselt number, hD_h/k

P pressure [bar]

Re Reynolds number, $u\rho D_h/\eta$

Sh Sherwood number, $gD_h/\rho^+ \mathbb{D}$

T absolute temperature [K]

t temperature [°C]

u component of velocity in the x-direction [m s^{-1}]

v component of velocity in the y-direction [m s^{-1}]

x, y coordinates [m].

Greek symbols

δ film thickness [m]

η dynamic viscosity [Pa s]

θ correction factor

ρ density [kg m^{-3}]

ρ^+ molar density [kmol m^{-3}].

Subscripts

a   critical interface condition

b   bulk

c   diffusional

i   condensate (or wall)/gas interface

n   noncondensables

sat   saturation

t   thermal

tot   total

v   vapour.

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differential equation, and applying equations (3) and (4), produces

$$c^+(y) = 1 + (c_i^+ - 1)\exp\left\{\frac{y}{\delta_c}\ln\left(\frac{1-c_b^+}{1-c_i^+}\right)\right\}. \quad (5)$$

This solution for the vapour mole fraction in a film was first obtained by Stefan [5].

The energy equation for the film, neglecting viscous dissipation, internal heat sources and radiation, is

$$\rho^+ c_p^+ v\frac{dt}{dy} + \rho^+ \mathbb{D}(c_{p,n}^+ - c_{p,v}^+)\frac{dc^+}{dy}\frac{dt}{dy} = k\frac{d^2t}{dy^2}. \quad (6)$$

The second term on the left-hand side represents the well-known Ackermann term. The boundary conditions on t are

Fig. 1. The film.

$$t(y = 0) = t_i \quad (7)$$

$$t(y = \delta_t) = t_b. \quad (8)$$

Substituting the relation between the mixture's molar specific heat and its composition

$$c_p^+ = c^+ c_{p,v}^+ + (1 - c^+)c_{p,n}^+ \quad (9)$$

and equations (2) and (5) into equation (6), solving the resulting equation and applying boundary conditions (7) and (8), yields the temperature in the film

$$t(y) = (t_b - t_i)\left[\frac{\exp\left\{\frac{c_{p,v}^+ y}{Le\,c_p^+ \delta_c}\ln\left(\frac{1-c_b^+}{1-c_i^+}\right)\right\} - 1}{\exp\left\{\frac{c_{p,v}^+ \delta_t}{Le\,c_p^+ \delta_c}\ln\left(\frac{1-c_b^+}{1-c_i^+}\right)\right\} - 1}\right] + t_i$$

$$(10)$$

where

$$Le = \frac{k}{\rho^+ c_p^+ \mathbb{D}}. \quad (11)$$

Equation (10) has been derived independently by Ackermann [6] and Colburn and Drew [7]. The latter authors also applied the film model expressions to convective heat and mass transfer in a closed channel. The bulk values of temperature and vapour mole fraction are then taken to be the mixed mean values, while δ_t and δ_c are taken to be D_h/Nu and D_h/Sh, where Nu and Sh denote the Nusselt and Sherwood number in the absence of mass transfer. In the next section these
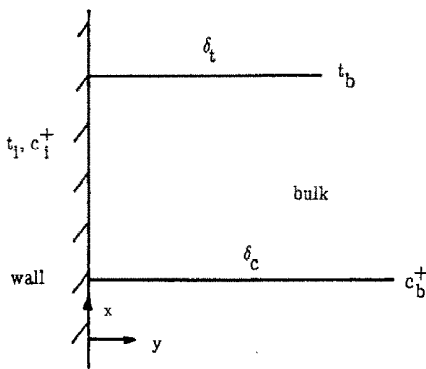
film model expressions are used to investigate supersaturation in the mixture.

3. THE FOG FORMATION CONDITION

In order to obtain a relation between c^+ and t in the film, which can then be compared with the saturation line, the coordinate y/δ_c is eliminated from equations (5) and (10), yielding

$$c^+ = G^+(t) = 1 + (c_i^+ - 1) \left(\frac{t - t_i}{t_b - t_i} \right)$$

$$\times \left(\exp \left\{ \frac{c_{p,v}^+ \delta_t}{c_p^+ Le \, \delta_c} \ln \left(\frac{1 - c_b^+}{1 - c_i^+} \right) \right\} - 1 \right) + 1 \right)^{(Le \, c_p^+)/c_{p,v}^+}.$$

$$(12)$$

This relation is a monotonically increasing function of t, since the first derivative of $G^+(t)$ with respect to t is positive

$$\frac{dG^+}{dt} = \frac{Le \, c_p^+}{c_{p,v}^+} \left(\frac{c_i^+ - 1}{t_b - t_i} \right)$$

$$\times \left(\exp \left\{ \frac{c_{p,v}^+ \delta_t}{c_p^+ Le \, \delta_c} \ln \left(\frac{1 - c_b^+}{1 - c_i^+} \right) \right\} - 1 \right) \left(\frac{t - t_i}{t_b - t_i} \right)$$

$$\times \left(\exp \left\{ \frac{c_{p,v}^+ \delta_t}{c_p^+ Le \, \delta_c} \ln \left(\frac{1 - c_b^+}{1 - c_i^+} \right) \right\} - 1 \right) + 1 \right)^{((Le \, c_p^+)/c_{p,v}^+) - 1}.$$

$$(13)$$

For $(Le \, c_p^+)/c_{p,v}^+ > 1$ the function $G^+(t)$ is concave, $G^+(t)$ is a straight line for $(Le \, c_p^+)/c_{p,v}^+ = 1$, while $G^+(t)$ is a convex curve for $(Le \, c_p^+)/c_{p,v}^+ < 1$. These properties follow from the second derivative of $G^+(t)$ with respect to t. The vapour mole fraction on the saturation line follows from

$$c^+ = F^+(t) = \frac{P_{v,sat}(t)}{P_{tot}}. \tag{14}$$

At the wall, denoting here the condensate/gas interface, $c_i^+ = F^+(t_i)$ prevails. On the basis of a consideration of vapour and temperature profiles, the following slope condition is suggested to examine whether supersaturation occurs in a condenser:

$$\left. \frac{\dfrac{dc^+}{dy} \Big|_{y=0}}{\dfrac{dt}{dy} \Big|_{y=0}} \right. = \frac{dG^+}{dt} \Big|_{t=t_i} = \frac{Le \, c_p^+}{c_{p,v}^+} \left(\frac{c_i^+ - 1}{t_b - t_i} \right)$$

$$\times \left(\exp \left\{ \frac{c_{p,v}^+ \delta_t}{c_p^+ Le \, \delta_c} \ln \left(\frac{1 - c_b^+}{1 - c_i^+} \right) \right\} - 1 \right) > \frac{dF^+}{dt} \Big|_{t=t_i}$$

$$(15)$$

which is based on the slopes of $F^+(t)$ and $G^+(t)$ in $t = t_i$. From this equation and the saturation con-

dition (implicitly excluding the possibility of supersaturation) fog formation can be detected.

The lowest permissible t_i at which fog is not yet formed, denoted as t_a, is obtained when equation (15) is an equality. Applying equation (13) and rewriting equation (15) then yields as the tangency condition

$$\frac{dF^+}{dt} \Big|_{t=t_a} = \frac{dG^+}{dt} \Big|_{t=t_a} = \frac{\delta_t}{\delta_c} \frac{\theta_c^+}{\theta_t^+} \frac{c_b^+ - c_a^+}{t_b - t_a} \tag{16}$$

where c_a^+ denotes the critical interface mole fraction ($c_a^+ = F(t_a)$). In this equation the thermal (Ackermann) correction factor is introduced as

$$\theta_t^+ = \frac{\dfrac{c_{p,v}^+ \delta_t}{Le \, c_p^+ \delta_c} \ln \left(\dfrac{1 - c_b^+}{1 - c_a^+} \right)}{\exp \left\{ \dfrac{c_{p,v}^+ \delta_t}{Le \, c_p^+ \delta_c} \ln \left(\dfrac{1 - c_b^+}{1 - c_a^+} \right) \right\} - 1} \tag{17}$$

and the diffusional mass transfer correction factor as

$$\theta_c^+ = \frac{\ln \left(1 - \dfrac{c_b^+ - c_a^+}{1 - c_a^+} \right)}{-\dfrac{c_b^+ - c_a^+}{1 - c_a^+}}. \tag{18}$$

Both these conventional film model correction factors can be found in Bird et al. [4], and are widely used in practice. For $t_i < t_a$, a part of the film is fogging ($t_i \leqslant t \leqslant t_a$), in this part t and c^+ are coupled by equation (14). In the superheated part ($t_a \leqslant t \leqslant t_b$) $c^+ = G^+(t)$ prevails. In Fig. 2 the physical principles of equations (15) and (16) are illustrated graphically. Johnstone et al. [1] were the first to employ the principle of the slope and tangency condition to assess fog formation. However, they used an incorrect expression, as will be explained below.

Their expression for the critical t_i ('equation (9)') is obtained when in equations (15)–(17) are substituted

$$\frac{\delta_t}{\delta_c} = \frac{Sh}{Nu} = Le^{1/3} \tag{19}$$

and

$$c_{p,v}^+ = c_p^+ \tag{20}$$

and

$$\theta_c^+ = 1. \tag{21}$$

Equation (19) is applicable to turbulent flow (the so-called Chilton–Colburn analogy) and to forced convective laminar flow in the entrance region of a channel. Johnstone et al. [1] experimentally examined laminar flow ($Re \cong 700$) of binary mixtures in this region of a circular tube. Approximation (20) implies that $c_{p,v}^+ = c_{p,n}^+$, see equation (9), and introduces an unacceptable inaccuracy. This is particularly the case with nitrogen and n-butyl alcohol mixtures (n-butyl alcohol, $c_{p,v}^+ \cong 135$ kJ kmol^{-1} K^{-1}; water, $c_{p,v}^+ \cong 34$ kJ kmol^{-1} K^{-1}; nitrogen, $c_{p,n}^+ \cong 28$ kJ kmol^{-1} K^{-1}).
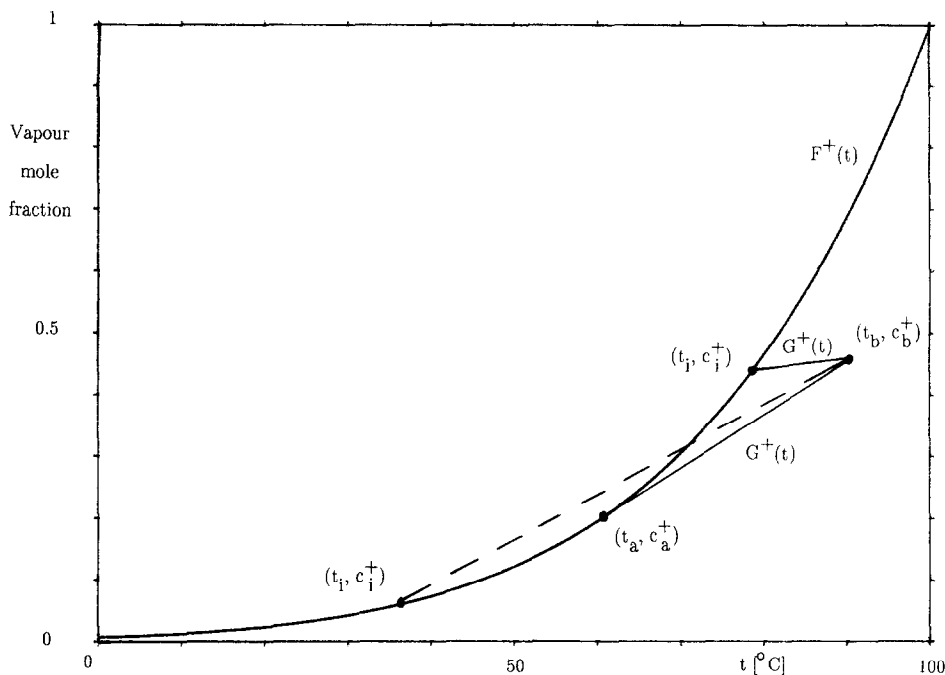
FIG. 2. Determination of t_a and prediction of fog formation (for $(Le \, c_p^+)/c_{p,v}^+ = 1$ and $\delta_c = \delta_t$).

Approximation (20) actually follows from a film model analysis where the Ackermann term in energy equation (6) is not taken into account. Assumption (21) is not correct either, since the diffusion correction factor θ_c^+ is of the same order of magnitude as the thermal correction factor; they are even identical when $(Le \, c_p^+)/c_{p,v}^+ = 1$. Equation (21) follows, in fact, from a film model analysis where the effect of the induced velocity on diffusion is neglected. The introduction of equations (20) and (21) might therefore be the reason why Johnstone et al. [1] observed discrepancies between some experiments and theory. In the next section these cases are discussed in some detail.

It is interesting to realize that the negative effect of equations (20) and (21) is less pronounced for mixtures with dilute vapour. To these mixtures the following applies; namely that $\theta_c^+ \cong 1$ and $\theta_t^+ \cong 1$, since $c_i^+ < c_b^+ \ll 1$. To determine fog formation and assess the boundary of the superheated and saturated regions in flowing mixtures of dilute wall condensing water vapour (with $\theta_c^+ = \theta_t^+ = 1$) in air, equations (15) and (16) have been applied fruitfully in linearized form by Toor [8, 9] (with $\delta_t/\delta_c = Sh/Nu = 1$), Koch [3] ($\delta_t/\delta_c = 1$) and Hayashi et al. [10] ($\delta_t/\delta_c = Le^{0.36} = 0.95$).

Using equation (15) (or equation (16)) fog can be detected. But on the other hand, if equation (15) is not fulfilled, superheating in the entire film is not necessarily guaranteed. This aspect of criterion (15) is discussed in the following. For $(Le \, c_p^+)/c_{p,v}^+ \geqslant 1$ superheating can be examined correctly with equation (15), since $G^+(t)$ is a straight line or a concave curve,

whereas $F^+(t)$ is convex. However, for $(Le \, c_p^+)/c_{p,v}^+ < 1$, it is theoretically possible that equation (15) predicts no fog formation, while both convex curves $F^+(t)$ and $G^+(t)$ intersect in the film. In Fig. 3 an example of such a supersaturation case in the film is depicted.

4. CONDENSATION EXPERIMENTS

The experimental results of Johnstone et al. [1] are now compared with the correct criterion for fog formation. We are particularly interested in the cases where fog was not predicted by the erroneous equations (16)–(21), though fog formation was observed for some situations. The introduction of equation (20) results namely in too high a value of t_a (since $c_{p,v}^+ > c_p^+ \, (> c_{p,n}^+)$), while assumption (21) causes too low a value of t_a (since $\Theta_c^+ > 1$), see equation (16). The net result of both introductions could be too low a value of t_a, resulting in an erroneous theoretical prediction of a superheated mixture and hence no fog formation. These situations were found only with mixtures of nitrogen and vapours of n-butyl alcohol and water, which are therefore treated here.

In Table 1 the experimental data for water vapour–nitrogen mixtures are listed, while in Table 2 those of n-butyl alcohol–nitrogen mixtures are summarized. In these tables t_a and the related predictions of Johnstone et al. [1] regarding fog formation are included. Using equations (16)–(19) the improved critical temperatures t_a are now re-determined. As the physical properties of air and nitrogen are very similar, the physical properties of air are used here, taken from
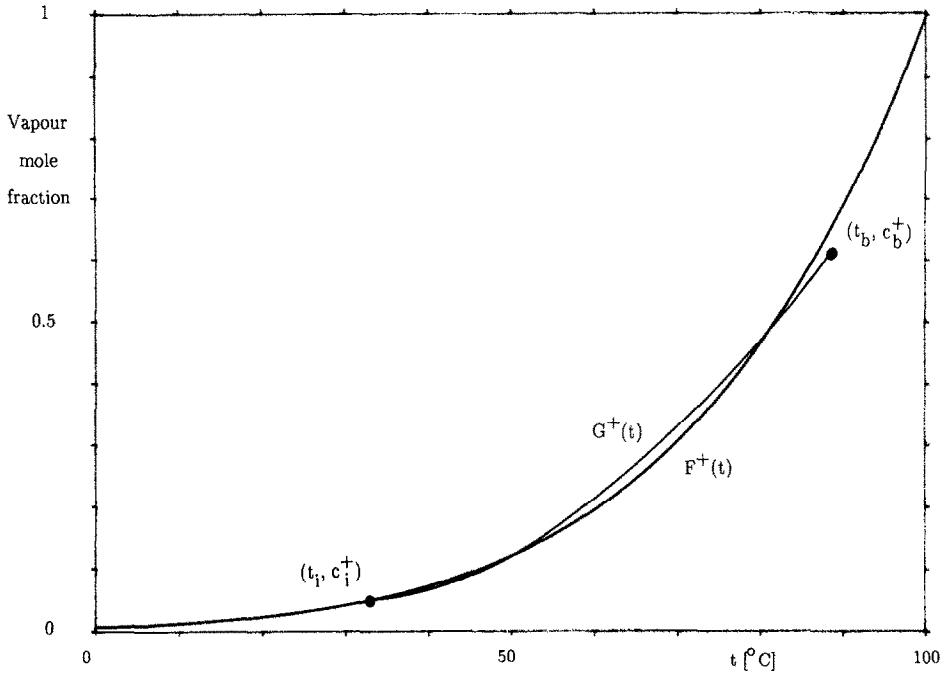
FIG. 3. Erroneous prediction of a superheated film with equation (15) for $(Le\, c_p^+)/c_{p,v}^+ < 1$ and $\delta_c = \delta_t$.

V.D.I. [11]. The latter is also used to supply the properties of water vapour. The diffusion coefficient of both gases is given by Edwards *et al.* [12] as

$$\mathbb{D}/[\text{m}^2\ \text{s}^{-1}] = 1.758 \times 10^{-4}\frac{(T/[\text{K}])^{1.685}}{P_{\text{tot}}/[\text{Pa}]}. \quad (22)$$

The molar specific heat $c_{p,v}^+$ of n-butyl alcohol is taken from Reid *et al.* [13], and the values of k and $\mathbb{D}$ of this vapour, provided by Landolt-Börnstein [14], are approximated by

$$k/[\text{W m}^{-1}\ \text{K}^{-1}] = 11.88 \times 10^{-3} + t/[^{\circ}\text{C}](5.54 \times 10^{-5}$$
$$+ 2.155 \times 10^{-7}t/[^{\circ}\text{C}]) \quad (23)$$

$$\mathbb{D}/[\text{m}^2\ \text{s}^{-1}] = 0.74 \times 10^{-5}\left(\frac{T/[\text{K}]}{273}\right)^{1.72}. \quad (24)$$

The molar densities ρ^+ of all the mixtures are simply determined using the ideal gas law, in combination with $P_{\text{tot}} = 1$ atm ($= 1.01325$ bar). The Antoine relations for the saturation pressures of both vapours stem from Reid *et al.* [13]. These vapour pressures yield saturation lines for both mixtures which correspond excellently to the lines drawn by Johnstone *et al.* [1] ('Fig. 2' and 'Fig. 3'). The saturation lines in Figs. 2 and 3 are, in fact, those of water vapour. The mixtures' molar specific heats are evaluated at the

Table 1. Dropwise wall condensation experiments of water vapour in nitrogen

| c_b^+ | t_b [°C] | Johnstone *et al.* [1] | | | | Equations (16)–(19) | | |
		t_i [°C]	t_a [°C]	Prediction	Observation	t_a [°C]	Prediction	Le
0.101	111.0	12.0	8.0	no fog	no fog	10.1	no fog	0.84
0.101	110.0	10.7	8.3	no fog	no fog	10.3	no fog	0.84
0.098	107.0	9.6	8.5	no fog	no fog	10.3	fog	0.84
0.097	105.0	8.3	8.3	critical value	no fog	10.4	fog	0.84
0.125	110.0	13.7	12.6	no fog	no fog	14.4	fog	0.83
0.123	111.0	12.4	12.4	critical value	fog	13.9	fog	0.83
0.123	112.5	12.0	12.0	critical value	fog	13.7	fog	0.83
0.095	123.5	11.1	5.0	no fog	no fog	7.0	no fog	0.84
0.100	134.0	6.0	4.5	no fog	no fog	6.5	fog	0.84
0.102	136.5	4.7	4.5	no fog	no fog	6.5	fog	0.84
0.096	135.0	4.5	3.7	no fog	fog	5.6	fog	0.84
0.169	129.0	16.5	15.0	no fog	no fog	17.0	fog	0.82
0.172	135.0	15.0	14.5	no fog	fog	16.3	fog	0.82
0.172	137.0	16.0	14.2	no fog	no fog	16.0	critical value	0.81
0.174	139.0	16.0	14.0	no fog	no fog	15.9	no fog	0.81
0.176	140.0	15.0	14.0	no fog	fog	16.0	fog	0.81
0.176	142.0	16.5	13.8	no fog	no fog	15.7	no fog	0.81

Table 2. Filmwise wall condensation experiments of n-butyl alcohol in nitrogen

		Johnstone et al. [1]				Equations (16) (19)		
c_b^+	t_b [C]	t_i [C]	t_a [C]	Prediction	Observation	t_a [C]	Prediction	Le
0.181	219.0	33.7	33.7	critical value	fog	31.0	no fog	1.41
0.178	229.0	35.0	32.7	no fog	no fog	29.9	no fog	1.42
0.180	265.0	31.0	30.2	no fog	no fog	27.3	no fog	1.41
0.180	138.0	56.6	44.0	no fog	no fog	41.3	no fog	1.43
0.180	152.0	43.9	41.5	no fog	fog	38.8	no fog	1.42
0.182	167.0	45.9	39.3	no fog	no fog	36.8	no fog	1.41
0.181	212.0	38.7	34.3	no fog	no fog	31.6	no fog	1.41
0.180	292.5	28.2	28.2	critical value	no fog	25.6	no fog	1.40
0.136	95.6	54.8	49.0	no fog	no fog	47.5	no fog	1.63
0.140	162.3	37.0	35.3	no fog	fog	33.5	no fog	1.59
0.136	174.0	34.7	33.3	no fog	fog	31.6	no fog	1.60
0.137	206.5	34.0	30.2	no fog	no fog	28.4	no fog	1.59
0.139	209.0	31.7	30.3	no fog	fog	28.2	no fog	1.58
0.139	210.0	30.3	30.3	critical value	fog	28.3	no fog	1.58
0.139	210.4	33.1	30.2	no fog	no fog	28.2	no fog	1.58
0.138	207.5	32.2	30.5	no fog	fog	28.4	no fog	1.58
0.139	207.5	32.8	30.6	no fog	no fog	28.5	no fog	1.58
0.147	229.0	32.6	29.5	no fog	no fog	27.4	no fog	1.54
0.149	250.0	29.1	28.0	no fog	no fog	26.0	no fog	1.53
0.153	250.0	28.8	28.3	no fog	no fog	26.3	no fog	1.51
0.147	220.8	31.4	30.2	no fog	fog	28.1	no fog	1.54
0.107	122.0	38.7	37.0	no fog	no fog	35.7	no fog	1.77
0.107	122.0	38.5	37.0	no fog	fog	35.7	no fog	1.77
0.110	128.0	42.1	36.3	no fog	no fog	35.1	no fog	1.75
0.106	129.0	38.7	35.3	no fog	fog	34.3	no fog	1.77
0.108	169.0	31.2	30.2	no fog	fog	28.9	no fog	1.75
0.109	180.5	34.4	29.0	no fog	no fog	27.8	no fog	1.74
0.110	190.0	32.9	28.0	no fog	no fog	26.9	no fog	1.73
0.108	195.0	33.7	27.5	no fog	no fog	26.2	no fog	1.74

bulk composition (see equation (9) with c^+ replaced by c_b^+). The mixtures' thermal conductivities are evaluated at the bulk composition, taking account of their composition following Perry and Green [15]

$$ k = \frac{c_b^+ k_v}{c_b^+ + \left(\dfrac{M_n}{M_v}\right)^{1/2}(1-c_b^+)} + \frac{(1-c_b^+)k_n}{1-c_b^+ + \left(\dfrac{M_v}{M_n}\right)^{1/2} c_b^+} \quad . $$

(25)

All the properties of the two components are evaluated at the bulk temperature. In Tables 1 and 2 the newly determined t_a and calculated Le are listed.

Table 1 reveals, owing to the effect of equation (21), that all the new t_a are larger than those of Johnstone et al. [1]. Accordingly, for most cases $t_i < t_a$ and hence fog formation is predicted, which is in agreement with the experimental observation. However, in a few situations fog is predicted but not observed. This can be explained from the difference between t_a and t_i, which is a measure of the degree of supersaturation. In general it follows that fog is not observed when t_i is slightly below t_a, while visible fog formation occurs when t_i exceeds the critical temperature t_a significantly. Table 1 also discloses that, according to the improved tangency condition, no inexplicable situations appear where a critical condition ($t_a = t_i$) or even superheating is predicted theoretically, but fog is observed experimentally.

The new results in Table 2 indicate, however, that the re-determined t_a are even lower than those of Johnstone et al. [1]. This is explained by the fact that $c_{p,v}^+/c_p^+$ is much larger than unity, typically $c_{p,v}^+/c_p^+ = 3.2$, for all the mixtures considered. This implies an enhancement of θ_i' and reduction of the calculated t_a, see equations (16) and (17). The observed fog can therefore not be explained with the aid of equations (16)–(19) derived here.

An intersection of $G^+(t)$ and $F^+(t)$ in the film, as depicted in Fig. 3, may result in fog formation as well. This intersection is possible since $(Le\, c_p^+)/c_{p,v}^+ < 1$ for both mixtures considered (water vapour–nitrogen, $(Le\, c_p^+)/c_{p,v}^+ \cong 0.7$; n-butyl alcohol–nitrogen, $(Le\, c_p^+)/c_{p,v}^+ \cong 0.5$), which implies that $G^+(t)$ is a convex curve. A numerical investigation of $G^+(t)$ and $F^+(t)$ with t ranging from t_i to t_b provided evidence for all the mixtures that an intersection of both curves does not occur when superheating was predicted by the slope condition. That is to say, $F^+(t) > G^+(t)$ in the entire film, thus predicted correctly by slope condition (15) or tangency condition (16). Summarizing, the experimental results obtained with n-butyl alcohol–nitrogen mixtures cannot be explained satisfactorily.

The major difference with the water vapour–nitrogen experiments, besides the higher temperatures and different Lewis numbers, is the filmwise condensation of n-butyl alcohol, since no promoter could be found for dropwise condensation. Oleic acid was used by

Johnstone *et al.* [1] to promote the dropwise condensation of water vapour. This dropwise condensation implies that the experimental interface temperature is better defined, which might be the reason overall agreement is found only between theory and water vapour–nitrogen experiments.

5. CONCLUSIONS

In this paper an improved tangency condition for predicting supersaturation in condensers has been derived. This condition is based on the slopes of the saturation line and film model expressions for the vapour mole fraction and temperature.

It has been demonstrated that supersaturation can be correctly predicted by this condition, irrespective of the value of $(Le\ c_p^+)/c_{p,v}^+$, but that for $(Le\ c_p^+)/c_{p,v}^+ < 1$ superheating in mixtures cannot be guaranteed. However, a thorough examination of temperature and vapour mole fraction profiles in various binary mixtures of nitrogen with vapours of water and n-butyl alcohol revealed that the condition correctly predicted superheating.

A comparison of the condition with fog formation experiments of Johnstone *et al.* [1], concerning dropwise wall condensation of water vapour in air, yields good agreement. If dropwise condensation of n-butyl alcohol had also been achieved by these investigators, complete agreement with theory might have been obtained here.

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UNE CONDITION TANGENTE AMELIOREE POUR LA FORMATION DE BROUILLARD DANS LES CONDENSEURS-REFROIDISSEURS

Résumé—En 1950 Johnstone *et al.* (*Ind. Engng Chem.* **42**, 2298–2302 (1950)) introduisirent la condition tangente pour déterminer la formation de brouillard (ou de buée) dans les mélanges binaires dans les canaux refroidis. Dans la présente analyse on montre que leur condition est erronée et une équation améliorée est obtenue. La condition est basée sur le transfert diffusionnel de chaleur et de masse à la paroi d'un condenseur et sur la pente de la ligne de saturation du composant vapeur à la température pariétale. On traite l'équation d'énergie et de diffusion dans un film stagnant près de la paroi (le modèle classique du film ou la théorie du film). La validité de la condition tangente améliorée est éprouvée avec les expériences de condensation pariétale faites par Johnstone *et al.* sur des mélanges d'azote avec des vapeurs d'eau et de n-butyl alcool et on constate un accord satisfaisant.

EINE VERBESSERTE BERÜHRUNGSBEDINGUNG ZUR BESTIMMUNG DER NEBELBILDUNG IN KÜHLER-KONDENSATOREN

Zusammenfassung—Im Jahr 1950 führten Johnstone *et al.* (*Ind. Engng Chem.* **42**, 2298–2302 (1950)) eine Berührungsbedingung ein, um die Nebelbildung binärer Gemische in gekühlten Kanälen zu bestimmen. In der vorliegenden Untersuchung wird gezeigt, daß die damals angegebene Bedingung fehlerhaft ist, weshalb eine verbesserte Gleichung abgeleitet wird. Die Bedingung basiert auf einer Beschreibung der Wärmestromdichte und Massenstromdichte durch Diffusion zur Kondensatorwand, sowie der Steigung der Sättigungskurve der Dampfkomponente bei Wandtemperatur. Die Stromdichten errechnen sich aus den vollständigen Transportgleichungen für Energie und Stoff in einem ruhenden Film in unmittelbarer Nähe der Wand (klassisches Filmmodell, bzw. klassische Filmtheorie). Die Gültigkeit der verbesserten Berührungsbedingung wird anhand der Experimente von Johnstone *et al.*, welche die Filmkondensation von Gemischen aus Stickstoff und Wasser- bzw. n-Butylalkohol-Dämpfen untersuchten, geprüft. Es ergibt sich eine zufriedenstellende Übereinstimmung.

ТАНГЕНЦИАЛЬНОЕ УСЛОВИЕ В ОХЛАДИТЕЛЯХ-КОНДЕНСАТОРАХ ПРИ ОБРАЗОВАНИИ ТУМАНА

Аннотация—В 1950 г. Джонстоун и др. (*Ind. Engng Chem.* **42**, 2298–2302 (1950)) ввели понятие тангенциального условия при образовании тумана (дымки) в бинарных смесях в охлаждаемых каналах. В настоящем исследовании показано, что предложенное условие является ошибочным, и выведено более адекватное выражение. Это условие содержит скорости теплопереноса и диффузионного массопереноса к стенке конденсатора, а также угол наклона линии насыщения пара при температуре стенки. Скорости переноса определены при помощи детального анализа уравнений сохранения энергии и диффузии в прилегающей к стенке неподвижной пленке (классические "модель пленки" и "теория пленки"). Эффективность использования рассматриваемого условия проверена путем сравнения с экспериментальными данными по конденсации на стенке, полученными Джонстоуном и др. для смесей азота с парами воды и н-бутилового спирта, при этом получено их удовлетворительное согласие.